CALCULATION OF HEAT TRANSFER IN A REGENERATOR WITH VARIABLE THERMOPHYSICAL PROPERTIES OF THE GAS AND CHECKERWORK

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The process of heat transfer in a regenerator in a transitional and steady-state regime is calculated with consideration of the temperature dependence of viscosity, density, thermal conductivity of the gas, and the specific heat of the checkerwork. A mathematical model of the operating conditions of a regenerator in a microcryogenic machine is developed.

The process of heat transfer in a regenerator was investigated analytically in [1-11]. A method of calculating the heating (cooling) of the checkerwork at the initial instant of operation of the regenerator is proposed in [1, 2, 6, 9]. The steady operating regime of a regenerator was investigated in [3-5, 7, 8]. The effect of a variable specific heat of the checkerwork on the efficiency of a regenerator in a steady regime was taken into account in [11].

The authors of the aforementioned studies considered heat transfer in a regenerator in a one-dimensional formulation. An arbitrary initial distribution of the temperature of the checkerwork over the length of the regenerator was assumed and the gas temperature at the entrance was assumed to be constant or an arbitrary function of time. In the equation of energy for the gas, the authors did not take into account, except in [8], the term characterizing the change of gas temperature with time, which in the given case cannot be neglected, since in regenerators of microcryogenic machines t and x/u are quantities of the same order. The pressure in the gas flow, and also the parameters characterizing the thermophysical properties of the gas and checkerwork were considered constant, except in [11].

The regenerator of a microcryogenic machine comprises a cylindrical tube filled with checkerwork in the form of a large number of closely adjacent layers of a fine metal mesh arranged normal to the x-axis, which is directed along the tube axis.

The following assumptions were made in calculating the heat-transfer process: a) heat conduction of the checkerwork is equal to zero along the gas flow and infinitely large normal to it; b) heat transfer by conduction by the gas and checkerwork and also by radiation is negligibly small; c) specific heat capacity and pressure of gas flow are constant; d) heat inflows from the ambient medium and gravitational forces are small.

Thus, heat-transfer processes in a regenerator can be described by a system of one-dimensional differential equations for parameters of the gas flow and checkerwork averaged over the cross section.

The energy equation for the gas is

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\alpha}{\rho c_p r_h}(T_c - T). \tag{1}$$

The energy equation for the checkerwork is

$$\frac{\partial T_c}{\partial t} = -\frac{\alpha}{\rho_c c_c r_h}\cdot\frac{m}{1-m}(T_c - T). \tag{2}$$

The equation of continuity is

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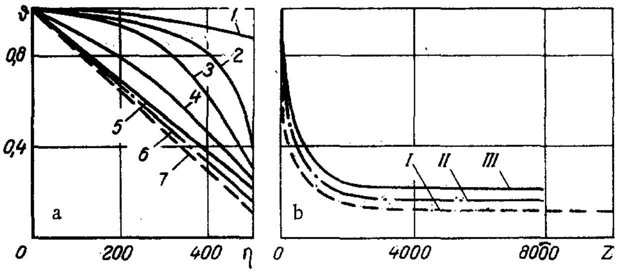

Fig. 1. Change of the dimensionless temperature of the checkerwork ϑ in the transitional regime. a) Over the length of the regenerator η: 1-4) characteristic distribution of the temperature of the checkerwork ϑ during cooling of the regenerator (the number of blowings Z increases accordingly from 0 to Z_r at which the regenerator arrives at a steady operating regime); 5-7) distribution of the temperature of the checkerwork in a steady-state regime: 5) $c_c = c_c(\vartheta)$, $\alpha = \alpha(\theta)$; 6) $c_c = c_c(\vartheta)$, $\alpha = \text{const}$; 7) $c_c = \text{const}$, $\alpha = \text{const}$; b) on the cold end of the regenerator: I) $c_c = \text{const}$, $\alpha = \text{const}$; II) $c_c = c_c(\vartheta)$, $\alpha = \text{const}$; III) $c_c = c_c(\vartheta)$, $\alpha = \alpha(\theta)$.

$$\frac{\partial}{\partial x}(\rho u) = 0. \tag{3}$$

Estimates show that the term $\partial\rho/\partial t$ in Eq. (3) need not be taken into account.

The equation of state for a perfect gas is

$$P = \rho RT. \tag{4}$$

The coefficient of heat transfer between the gas and checkerwork was calculated by the formula obtained in [12]:

$$\mathrm{St}\,\mathrm{Pr} = A(\mathrm{Re})^{-n}, \tag{5}$$

from where

$$\alpha = Ad_h^{-n}(\rho u)^{1-n}\frac{\lambda}{\mu^{1-n}}. \tag{6}$$

The temperature dependence of viscosity and thermal conductivity is expressed by the Sutherland equation [14]:

$$\mu_T = \mu_0\frac{273+C}{T+C}\left(\frac{T}{273}\right)^{3/2}, \tag{7}$$

$$\lambda_T = \lambda_0\frac{273+C}{T+C}\left(\frac{T}{273}\right)^{3/2}. \tag{8}$$

The temperature dependence of the specific heat of the checkerwork material [14] is approximated by a power function of the form

$$C_c = aT_c^{\gamma}. \tag{9}$$

After transformations of Eqs. (1)-(4) with consideration of (6)-(9), we obtain a system of partial differential equations describing the change of temperature of the checkerwork and gas:

$$\rho\frac{\partial T}{\partial t} + \rho u\frac{\partial T}{\partial x} = \frac{\alpha_0}{r_h c_P}B(T_c - T), \tag{10}$$

$$\frac{\partial T_c}{\partial t} = -\frac{\alpha_0}{\rho_c r_h a}\cdot\frac{m}{1-m}B\frac{1}{T_c^{\gamma}}(T_c - T), \tag{11}$$

$$P = \rho RT, \tag{12}$$

where

$$\alpha_0 = Ad_h^{-n}(\rho u)^{1-n}\frac{\lambda_0}{\mu_0^{1-n}};$$

$$B = \left(\frac{273+C}{T+C}\right)^n\left(\frac{T}{273}\right)^{3n/2}.$$

Introducing the relative variable

$$\theta = \frac{T}{T_0};\ \vartheta = \frac{T_c}{T_0};\ \tau = \frac{t}{r_h\,\mathrm{St}\,u_0};\ \eta = \frac{x}{r_h/\mathrm{St}},$$

we transform Eqs. (10)-(12) to the dimensionless form:

$$\frac{1}{\theta}\frac{\partial\theta}{\partial\tau} + \frac{\partial\theta}{\partial\eta} = \chi(\vartheta - \theta), \tag{13}$$

$$\frac{\partial\vartheta}{\partial\tau} = -N\chi\frac{1}{\vartheta^{\gamma}}(\vartheta - \theta), \tag{14}$$

where

$$\chi = \left(\frac{273+C}{\theta T_0 + C}\right)^n\left(\frac{T_0}{273}\right)^{3n/2}\theta^{3n/2};$$

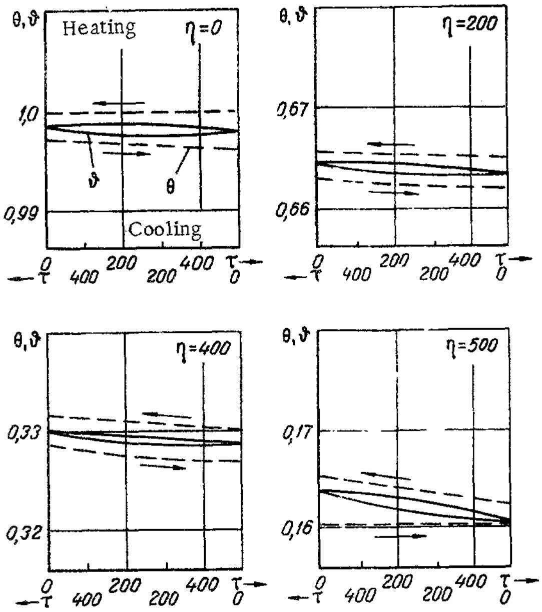

Fig. 2. Change of the dimensionless temperature of the gas θ and checkerwork ϑ during the cycle (heating and cooling) in four sections of the regenerator.

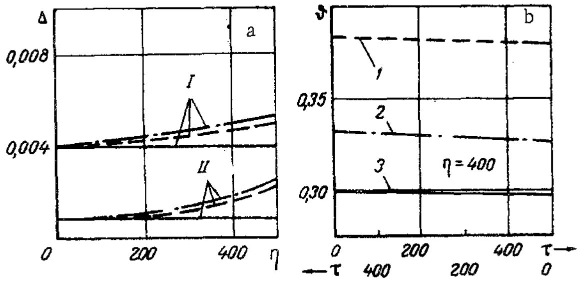

Fig. 3. Distribution of the difference of temperature of the gas θ and checkerwork ϑ over the length of the regenerator η in the steady regime. a: I) $\Delta = \theta_{ho} - \theta_{co}$, II) $\Delta = \vartheta_{ho} - \vartheta_{co}$, and change of temperature of the checkerwork during the cycle in the steady regime; b: 3) $c_c = \text{const}$, $\alpha = \text{const}$; 2) $c_c = c_c(\vartheta)$, $\alpha = \text{const}$; 1) $c_c = c_c(\vartheta)$, $\alpha = \alpha(\theta)$.

$$N = \frac{\rho c_p m}{\rho_c c_{c_0}(1-m)},$$

$\rho c_p m$ and $\rho_c c_{c_0}(1-m)$ are the volume specific heats of the gas and checkerwork, respectively.

The boundary conditions are:

$$\theta(0,\ \eta) = \theta_1(\eta),\ \theta(\tau,\ 0) = \theta_2(\tau),\ \vartheta(0,\ \eta) = \vartheta_1(\eta),\quad 0 \leqslant \tau \leqslant \tau_1;\ 0 \leqslant \eta \leqslant \eta_1. \tag{15}$$

The system of partial differential equations (13)-(14) together with boundary conditions (15) is solved by the method of finite differences [13] on the M-20 computer. For this purpose, the system of equations (13)-(14) is approximated by a system of difference equations (explicit scheme)

$$\frac{1}{\theta_n^m}\frac{\theta_n^{m+1}-\theta_n^m}{k} + \frac{\theta_n^m-\theta_{n-1}^m}{h} = \chi_n^m(\vartheta_n^{m+1}-\theta_n^{m+1}), \tag{16}$$

$$\frac{\vartheta_n^{m+1}-\vartheta_n^m}{k} = -N\chi_n^m\frac{1}{(\vartheta_n^m)^\gamma}(\vartheta_n^{m+1}-\theta_n^{m+1}),$$

$$\theta_n^0 = \theta_1(j),\ \theta_0^m = \theta_2(i),\ \vartheta_n^0 = \vartheta_1(j),\ 0 \leqslant \tau \leqslant 5\cdot10^2,\ 0 \leqslant \eta \leqslant 5\cdot10^2. \tag{17}$$

We investigated the process of heat transfer in the regenerator as the machine arrived at the regime. Therefore, we take as initial conditions the distribution of the temperature of the checkerwork over the length of the regenerator which is established naturally in the nonoperating microcryogenic machine, i.e., the temperature of the checkerwork is constant and equal to the temperature of the ambient medium ϑ_a. It is assumed that during heating of the checkerwork the gas is blown with temperature $\theta_1 = \vartheta_a + \Delta\vartheta$, where $\Delta\vartheta$ is incomplete recuperation in the condenser (thereby simulating the presence of a compressor–condenser circuit), whilst the temperature of the gas at the entrance of the regenerator remains constant during blowing. After the end of heating the checkerwork, blowing of the regenerator with cold gas with temperature $\theta_2 = \varepsilon\theta'$ begins (θ' is the temperature of the gas on the cold end of the regenerator at the end of warm blowing; ε is the degree of decrease of the temperature of the gas flow simulating the compressed-gas motor–heater circuit). Multiple repetition of the described processes of heating and cooling of the checkerwork permits an approximate reproduction of real operating conditions of a regenerator in a microcryogenic machine.

The calculation was made for helium for $N = 1.3 \cdot 10^{-3}$, $1.86 \cdot 10^{-3}$; $\gamma = 0.64$, 0; $A = 0.63$; $n = 0.45$; $T_0 = 300$°K; $\varepsilon = 0.97$.

When the regenerator arrives at the regime the temperature of the checkerwork on its cold end (Fig. 1a) drops rapidly. Then cooling gradually extends to the warm end of the regenerator. The rate of cooling of the checkerwork depends on the temperature drop ε on the cold end of the regenerator. Larger values of ε allow a lower temperature to be obtained and a reduction of the time to arrive at the required temperature regime, but increase the losses.

The change of the temperature of the checkerwork on the cold end of the regenerator in the transient process is shown in Fig. 1b for the cases: $c_c = c_c(\vartheta)$, α = const ($N = 1.3 \cdot 10^{-3}$, $\gamma = 0.64$, $\chi = 1$); c_c = const, α = const ($N = 1.86 \cdot 10^{-3}$, $\gamma = 0$, $\chi = 1$); and $c_c = c_c(\vartheta)$, $\alpha = \alpha(\theta)$ ($N = 1.3 \cdot 10^{-3}$, $\gamma = 0.64$, $\chi = \chi(\theta)$). A decrease in the specific heat of the checkerwork and heat-transfer coefficient with decrease in temperature has a substantial effect on heat transfer in the regenerator and on the value of the final temperature which can be attained on the cold end of the regenerator, their effect on heat transfer being identical. The use of the average value of the specific heat of the checkerwork rather than the initial value as c_{c_0} does not lead to any detectable changes of the temperature of the checkerwork (an appropriate calculation was made).

A linear distribution of the temperature of the checkerwork over the length of the regenerator is observed in the steady regime (Fig. 1a). The temperature of the checkerwork describes a characteristic hysteresis loop, whereby the average temperature difference $\Delta\vartheta_{av} = (\vartheta_{ho} - \vartheta_{co})$ has a maximum value at the ends of the regenerator (Fig. 2). However, this difference is small and the change of the temperature of the checkerwork during heating (cooling) can be considered linear.

The difference of temperature of the gas and checkerwork during heating (cooling) for c_c = const and α = const (Fig. 3a) is constant over the length of the regenerator (on the assumption of no heat inflows from the external medium), but for $c_c = c_c(\vartheta)$ and $\alpha = \alpha(\theta)$ it increases toward the cold end of the regenerator. When c_c = const and α = const the temperature of the checkerwork (Fig. 3b) fluctuates in a narrower range during the cycle.

The calculations of heat transfer in the regenerator of a microcryogenic machine showed that the rate of arrival at the regime depends considerably on the temperature difference on the cold end of the regenerator; the distribution of the temperature of the checkerwork over the length of the regenerator can be considered linear, including also its ends [5]. In this case the average temperature difference of the checkerwork during heating and cooling is insignificant, and therefore we can consider that the temperature varies linearly with time; a decrease of the specific heat of the checkerwork and heat-transfer coefficient with a decrease of temperature accelerates the arrival of the regenerator at the regime and introduces a substantial correction into the determination of the final temperature on the cold end of the regenerator in the transitional regime in comparison with the calculation with constant average (during the process) values of the thermophysical characteristics of the gas and checkerwork.

NOTATION

T, T_c	are the temperatures of the gas flow and checkerwork, °K;
θ, ϑ	are the dimensionless temperature of the gas flow and checkerwork;
t	is the time, sec;
x	is the length of regenerator, m;
τ, η	are the dimensionless time and length;
P	is the pressure of gas flow, N/m^2;
u	is the velocity of gas flow, m/sec;
ρ	is the gas density, kg/m^3;
ρ_c	is the density of checkerwork material, kg/m^3;
μ	is the gas viscosity, N·sec/m^2;
λ	is the thermal conductivity of gas, W/m·deg;
c_P	is the specific heat of gas at P = const, J/kg·deg;
R	is the gas constant, J/kg·deg;
St	is the Stanton number, dimensionless heat-transfer coefficient;
Pr	is the Prandtl number;
Re	is the Reynolds number;
r_h	is the hydraulic radius, m;
m	is the porosity;
α	is the heat transfer coefficient, W/m^2·deg;
c_c	is the specific heat of checkerwork J/kg·deg.

Subscripts

0 initial;
ho hot;
co cold.

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